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NEUTRINO REACTIONS ON NUCLEAR TARGETS ‡

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Abstract: We examine the systematics of deep inelastic neutrino scattering from complex nuclei by computing the cross section for quasi-elastic scattering and for quasi-free resonance production. We retain relativistic kinematics for the recoiling particle and the full relativistic hadronic weak vertex. The isobar cross section is expressed in terms of helicity amplitudes of the weak current, defined through an application of the Jacob-Wick formalism to the general isobar-nucleon weak vertex. The cross section is computed analytically for the nuclear Fermi gas model. We stress that exactly the same model has already been very successfully applied to inelastic electron scattering from complex nuclei.

1. INTRODUCTION

With the emergence of medium and high energy accelerators capable of producing high-intensity neutrino beams, neutrino reactions on nuclear targets can be studied experimentally with much greater precision than was previously possible. Such processes can provide information both on the dynamics of nuclear systems and on the weak interactions of elementary particles. First, the neutrino represents a new probe with which the nuclear physicist can study the structure of complex nuclei. In particular, the conserved vector current hypothesis, which identifies the isovector hadronic electromagnetic current and the hadronic weak vector currents as an isotriplet, implies that neutrino reactions can furnish both the vector and axial vector nuclear current densities when combined with the appropriate electron scattering measurements. For the particle physicist, neutrino induced processes offer the only presently feasible method for investigating the weak interaction at high energy, and nuclear targets are certain to find wide use in these investigations in order to enhance the counting rates. A reliable theory of the nuclear structure effects is essential for extracting the "elementary" neutrino-nucleon amplitudes from experimental data.

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We examine the systematics of deep inelastic neutrino scattering‡ from complex nuclei, within the framework of the nuclear Fermi gas model. Our calculation is an extension of earlier work on inelastic electron scattering [1] and assumes that the large momentum transfer, large energy loss part of the cross section corresponds roughly to "quasi-free" scattering; i.e., to direct collisions with the individual nucleons in the nucleus. We make a first attempt to compute the neutrino cross section in the region of lepton energy loss corresponding to excitation of the (3 - 3) resonance by assuming that quasi-free N^* excitation dominates single pion production. The quasi-elastic scattering is computed in the same model.

Other authors have calculated the neutrino quasi-elastic cross section. Goulard and Primakoff [2] use SU(4) supermultiplet theory and the closure approximation to calculate $d\sigma/d\Omega$. In their model, the nucleons in the ground state of the target are treated non-relativistically in the impulse approximation, while the recoil nucleon is relativistic. They employ a non-relativistic reduction of the nuclear vertex. Piketty [3] considers target states comprised of either non-relativistic or relativistic nucleons for large nuclei and uses the nuclear shell model for light nuclei. The ejected nucleon is treated relativistically. These calculations include a nuclear absorption factor for the recoiling nucleon. In sect. 5 we argue, as Løvseth [4] has, that this factor should not be included when only the final lepton is observed. York-Peng Yao [5] derives an analytic expression for quasi-elastic scattering from a relativistic Fermi gas by averaging single nucleon cross sections appropriate for parallel nucleon and neutrino momenta over all nucleon velocities. As this author himself points out, there is an ambiguity in this procedure. We sum the square of the matrix element over the nuclear states. Our results do not quite agree with those obtained by Yao, even in the limit of zero binding energy; however, we feel that ours is a more consistent procedure. Løvseth [4] has computed both electron and neutrino quasi-elastic cross sections on target states with realistic momentum distributions. He uses relativistic kinematics and amplitudes. Bell and Llewellyn-Smith [6] compute $d\sigma/d\Omega$ using non-relativistic target nuclear shell wave functions and a recoil factor corresponding to scattering from a stationary nucleon. The amplitude is obtained by reduction of the relativistic amplitude.

Our work is motivated by the very successful application of the same model to quasi-elastic electron scattering on a wide variety of nuclear targets [7] and by the general agreement with the isobar peaks revealed by the recent high energy electron scattering experiments of Titov et al. [8]. Our neutrino quasi-elastic scattering and isobar excitation results complement the work on electron scattering and allow us to look at the four related processes all in the same model. As in the earlier work, we use relativistic kinematics and the full relativistic weak interaction vertex for both neutrino processes. The target nucleon wave functions are taken to be plane waves

‡ Neutrino scattering is taken to mean $\nu_\ell(\bar{\nu}_\ell) \rightarrow \ell^-(\ell^+)$ and "deep inelastic" implies that the lepton energy loss is large compared to the energy needed for nuclear breakup.

with energies lowered from free particle energies by an average nuclear potential. Although this choice of wave functions corresponds to a nucleus without detailed structure, the dominant features of the nuclear cross section are consistently represented.

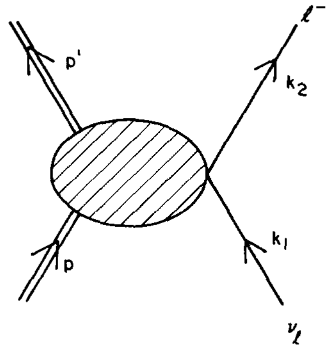
The parameters of nuclear Fermi momentum and potential depth may also be determined from electron scattering experiments. The excellent agreement of the electron scattering results [7] suggests that our simple calculation should provide a reliable estimate of the weak interaction response function in the region of quasi-elastic scattering and resonance production.

An understanding of the quasi-elastic and N^* peaks is interesting for several reasons. The quasi-elastic peak directly measures the single-particle structure of the nucleus, and neutrino (antineutrino) scattering can be used to provide a dynamical determination [7, 9] of the neutron (proton) Fermi momentum and average interaction energy as a function of atomic number. One would then like to use the knowledge of the quasi-elastic peak to "remove" the nuclear physics and thus to learn something about the creation and propagation of nucleon isobars in the nuclear medium. Finally, separation of the strictly nuclear effects from meson production is in itself important. For example, this separation is required in testing Bell's prediction of a nuclear shadow effect in forward neutrino reactions [10] and, because of the energy spread in neutrino beams, in extracting nucleon form factors form nuclear scattering experiments [11]. This lack of good energy resolution does limit the usefulness of neutrino beams in nuclear physics investigations; our calculation has the advantage of yielding an analytic expression for the neutrino cross section, so it may be readily averaged over any real neutrino spectrum.

In sect. 2 we derive the general cross section for neutrino scattering from nuclei in terms of the nuclear response function. In sect. 3 we relate the response function to the single nucleon amplitudes. We apply the helicity analysis of Jacob and Wick [12] to a study of an arbitrary isobar-nucleon weak vertex, which allows us to express the response function for quasi-free excitation of any isobar directly in terms of helicity amplitudes of the vector and axial currents. Numerical results using parameters determined from electron scattering are presented in sect. 4. In sect. 5 we discuss the validity of the model and summarize the main results.

2. GENERAL RESULTS FOR NEUTRINO SCATTERING

We consider the process illustrated in fig. 1, in which a charged lepton of mass m_ℓ is detected at an angle θ with respect to the incident neutrino. From the current-current form of the interaction, the usual Feynman rules yield the cross section



$q = p - p' = k_2 - k_1$

Fig. 1. Neutrino scattering on a nuclear target.

$$d^2\sigma = \frac{G^2}{2}\,\frac{1}{(2\pi)^2}\,\frac{1}{2\,|k_1\cdot p|}\,\frac{d\boldsymbol{k}_2}{2\epsilon_2}\,\eta_{\mu\nu}\,W_{\mu\nu}\ ,$$

$$\eta_{\mu\nu} = \mathrm{Tr}\,\{\gamma_\mu(1+\gamma_5)\not{k}_1\gamma_\nu(1+\gamma_5)\not{k}_2\}\ ,$$

$$W_{\mu\nu} = (2\pi)^3\,\Omega\,\bar{\sum_i}\sum_f\,\delta^{(4)}(p-p'-q)\langle p\,|\,J_\nu^{(-)}(0)|\,p'\rangle\,\langle p'|\,J_\mu^{(+)}(0)|\,p\rangle E\ ; \qquad (1)$$

$J_\mu^{(\pm)}(0) = V_\mu^{(\pm)}(0) + A_\mu^{(\pm)}(0)$ is the appropriate isospin component of the nuclear weak current, Ω is the quantization volume, E is the energy of the target, and the weak coupling constant is $G = 1.023\times 10^{-5}/m^2$, for m the proton mass ‡.

Lorentz and time reversal invariance restrict $W_{\mu\nu}$ to the form

$$W_{\mu\nu} = W_1\delta_{\mu\nu} + W_2/m_\mathrm{T}^2\,p_\mu\,p_\nu + W_\alpha/m_\mathrm{T}^2\,q_\mu q_\nu$$
$$+\,W_\beta/m_\mathrm{T}^2(p_\mu q_\nu + p_\nu q_\mu) + W_8/m_\mathrm{T}^2\,\epsilon_{\mu\nu\sigma\tau}\,p_\sigma q_\tau\ , \qquad (2)$$

where m_T is the target mass. The form factors W_j depend only upon the scalars q^2 and $q\cdot p$, and the last term corresponds to vector-axial interference. With the definition $\cos\chi = k_2/\epsilon_2\cos\theta$, the lab cross section is

$$\left(\frac{d^2\sigma}{dk_2\,d\Omega_2}\right)_\nu = \frac{G^2\,k_2^2\cos^2(\tfrac{1}{2}\chi)}{2\pi^2\,m_\mathrm{T}}\,\Big\{W_2 + [\,2W_1 + \frac{m_\ell^2}{m_\mathrm{T}^2}\,W_\alpha]\tan^2(\tfrac{1}{2}\chi)$$
$$+\,(W_\beta + W_8)\,m_\ell^2/(m_\mathrm{T}\,\epsilon_2\cos^2(\tfrac{1}{2}\chi))$$
$$-\,2W_8/m_\mathrm{T}\tan(\tfrac{1}{2}\chi)\sec(\tfrac{1}{2}\chi)\,[q^2\cos^2(\tfrac{1}{2}\chi) + |\boldsymbol{q}|^2\sin^2(\tfrac{1}{2}\chi) + m_\ell^2]^{\frac{1}{2}}\Big\}\ . \qquad (3)$$

‡ There is insuficicient experimental evidence for this process to warrant inclusion of the Cabibbo angle in the weak coupling constant. We simply ignore it.

For antineutrino reactions the sign of W_8 is reversed. Dropping the time reversal requirement for the nuclear currents would permit a term in $W_{\mu\nu}$ proportional to $(p_\mu q_\nu - p_\nu q_\mu)$, but this would not contribute to the cross section when contracted with $\eta_{\mu\nu}$.

3. QUASI-FREE PROCESSES

For quasi-free processes the nuclear weak current is obtained by summing the individual nucleon currents:

$$J_\mu^{(\pm)}(0) = \sum_{\boldsymbol{k},\boldsymbol{k}',\lambda,\lambda'} a^\dagger_{\boldsymbol{k}'\lambda'} \langle \boldsymbol{k}'\lambda' | j_\mu^{(\pm)}(0) | \boldsymbol{k}\lambda\rangle a_{\boldsymbol{k}\lambda} \qquad \text{(quasi-elastic)}$$

$$= \sum_{\boldsymbol{k},\boldsymbol{k}',\lambda,\lambda'} b^\dagger_{\boldsymbol{k}'\lambda'} \langle \boldsymbol{k}'\lambda' | j_\mu^{(\pm)}(0) | \boldsymbol{k}\lambda\rangle a_{\boldsymbol{k}\lambda} \qquad (\mathrm{N}^* \text{ production}) , \tag{4}$$

with $a^\dagger(b^\dagger)$ the free nucleon (isobar) creation operator. The elementary current $j_\mu^{(+)}(j_\mu^{(-)})$ connects a neutron (proton) to a proton (neutron) or an isobar. We describe the target state as a superposition of non-interacting neutron and proton Fermi gases with momentum distributions $n_\mathrm{n}(k)$ and $n_\mathrm{p}(k)$ respectively and can now express $W_{\mu\nu}$ in the lab frame in terms of the single nucleon matrix elements:

$$(W_{\mu\nu})_{\mathrm{lab}} = \int \mathrm{d}\boldsymbol{k}\, f(\boldsymbol{k},\boldsymbol{q},\omega)\, T_{\mu\nu} , \tag{5a}$$

$$f(\boldsymbol{k},\boldsymbol{q},\omega) = \frac{m_\mathrm{T}\Omega}{(2\pi)^3} \frac{\delta(\epsilon_k - \epsilon_{k-q} + \omega) n_\mathrm{i}(k)(1 - n_\mathrm{f}(|\boldsymbol{k}-\boldsymbol{q}|))}{\epsilon_k \epsilon_{k-q}} , \tag{5b}$$

$$T_{\mu\nu} = \epsilon_k \epsilon_{k-q} \Omega^2 \sum_{\lambda\lambda'} \langle \boldsymbol{k}-\boldsymbol{q}\lambda' | j_\mu^{(+)}(0) | \boldsymbol{k}\lambda\rangle \langle \boldsymbol{k}\lambda | j_\nu^{(-)}(0) | \boldsymbol{k}-\boldsymbol{q}\lambda'\rangle \tag{5c}$$

$$\equiv T_1 \delta_{\mu\nu} + T_2/m^2\, k_\mu k_\nu + T_\alpha/m^2\, q_\mu q_\nu + T_\beta/m^2 (k_\mu q_\nu + k_\nu q_\mu)$$
$$+ T_8/m^2\, \epsilon_{\mu\nu\sigma\tau} k_\sigma q_\tau . \tag{5d}$$

The single particle form factors depend only upon q^2, and $n_\mathrm{i}(k)$ is the neutron (proton) momentum distribution for incident neutrinos (antineutrinos): clearly these roles are reversed for $n_\mathrm{f}(k)$. We include the Pauli exclusion factor $(1 - n_\mathrm{f}(|\boldsymbol{k}-\boldsymbol{q}|))$ for quasi-elastic scattering; for a pure Fermi gas model $n_\mathrm{i}(k) = \theta(k_{\mathrm{F}_\mathrm{i}} - |\boldsymbol{k}|)$. This factor simply ensures that the recoil nucleon lies outside the Fermi sea. For a generalized momentum distribution this factor can only be approximate and, as noted by Løvseth [5], we must require that $n(k) \leq 1$ for (5b) to be meaningful.

The Lorentz transformation properties of $W_{\mu\nu}$ and $T_{\mu\nu}$ are now used to determine the W_j in terms of the T_j. In the lab frame, (3) becomes

$$W_{\mu\nu} = W_1\delta_{\mu\nu} - W_2\delta_{\mu 4}\delta_{\nu 4} + W_\alpha/m_T^2 q_\mu q_\nu$$
$$+ iW_\beta/m_T(\delta_{\mu 4}q_\nu + \delta_{\nu 4}q_\mu) - iW_8/m_T\epsilon_{\mu\nu\sigma 4}q_\sigma\ . \tag{6}$$

Each term in (5d) is inserted into (5a) and then compared with (6). In performing the angular integration, τ is the angle between $\boldsymbol{k}$ and $\boldsymbol{q}$ determined by the delta function. The result of this computation is

$$W_1 = a_1 T_1 + \tfrac{1}{2}(a_2 - a_3)\, T_2\ ,$$
$$W_2 = [a_4 + 2\omega/|\boldsymbol{q}|a_5 + \omega^2/|\boldsymbol{q}|^2 a_3 + \tfrac{1}{2}q^2/|\boldsymbol{q}|^2(a_2 - a_3)]\, T_2\ ,$$
$$W_\alpha = m_T^2/|\boldsymbol{q}|^2(\tfrac{3}{2}a_3 - \tfrac{1}{2}a_2)\, T_2 + m_T^2/m^2\, a_1 T_\alpha + 2m_T^2/(m|\boldsymbol{q}|)\, a_6 T_\beta\ ,$$
$$W_\beta = m_T/m\,(a_7 + \omega/|\boldsymbol{q}|a_6)\, T_\beta\ ,$$
$$W_8 = m_T/m\,(a_7 + \omega/|\boldsymbol{q}|a_6)\, T_8\ , \tag{7}$$

$$a_1 = \int \mathrm{d}\boldsymbol{k} f(\boldsymbol{k},\boldsymbol{q},\omega)\ , \qquad a_5 = \int \mathrm{d}\boldsymbol{k} f(\boldsymbol{k},\boldsymbol{q},\omega)\,\frac{\epsilon_k k\cos\tau}{m^2}\ ,$$
$$a_2 = \int \mathrm{d}\boldsymbol{k} f(\boldsymbol{k},\boldsymbol{q},\omega)\,\frac{k^2}{m^2}\ , \qquad a_6 = \int \mathrm{d}\boldsymbol{k} f(\boldsymbol{k},\boldsymbol{q},\omega)\,\frac{k\cos\tau}{m}\ ,$$
$$a_3 = \int \mathrm{d}\boldsymbol{k} f(\boldsymbol{k},\boldsymbol{q},\omega)\,\frac{k^2\cos^2\tau}{m^2}\ , \qquad a_7 = \int \mathrm{d}\boldsymbol{k} f(\boldsymbol{k},\boldsymbol{q},\omega)\,\frac{\epsilon_k}{m}\ ,$$
$$a_4 = \int \mathrm{d}\boldsymbol{k} f(\boldsymbol{k},\boldsymbol{q},\omega)\,\frac{\epsilon_k}{m^2}\ .$$

The a_j contain all the nuclear physics in the single particle momentum distributions and energies and can be evaluated analytically for a simple Fermi gas model $n_i(k) = \theta(k_{F_i} - |\boldsymbol{k}|)$. These results are given in the appendix. We now evaluate T_j for quasi-elastic scattering and resonance production in terms of "elementary" form factors.

3.1. *Quasi-elastic scattering*

For quasi-elastic scattering the current matrix element is

$$\langle \boldsymbol{k}'\lambda'|j_\mu^{(\pm)}(0)|\boldsymbol{k}\lambda\rangle = i\left(\frac{m^2}{\epsilon_k\epsilon_{k'}\Omega^2}\right)^{\frac{1}{2}} \bar{u}(\boldsymbol{k}'\lambda')\,\{F_1\gamma_\mu + F_2\sigma_{\mu\alpha}q_\alpha$$
$$- iF_S q_\mu\tau_z + F_A\gamma_5\gamma_\mu - iF_P\gamma_5 q_\mu + F_T\gamma_5\sigma_{\mu\alpha}q_\alpha\tau_z\}\,\tau_\pm u(\boldsymbol{k}\lambda)\ , \tag{8}$$

where we have kept the second-class currents F_S and F_T for generality (note that CVC is invalid if $F_S(q^2) \neq 0$). All F_i are real as a consequence of hermiticity and time reversal invariance. Inserting (8) into (5c) and carrying out the resulting traces yields the T_j:

$$T_1 = \tfrac{1}{2}q^2(F_1 + 2mF_2)^2 + (2m^2 + \tfrac{1}{2}q^2)F_A^2 ,$$

$$T_2 = 2m^2(F_1^2 + q^2 F_2^2 + F_A^2 + q^2 F_T^2) ,$$

$$T_\alpha = -m^2/q^2\, T_1 + \tfrac{1}{4}T_2 + m^2 F_S[-2mF_1 + q^2 F_2 + (2m^2 + \tfrac{1}{2}q^2)F_S]$$
$$+ m^2(2mF_A - q^2 F_P)[-F_T + 1/(2q^2)(2mF_A - q^2 F_P)] ,$$

$$T_\beta = -\tfrac{1}{2}T_2 + m^2 F_S[2mF_1 - q^2 F_2] + m^2 F_T[2mF_A - q^2 F_P] ,$$

$$T_8 = 2m^2 F_A(F_1 + 2mF_2) . \tag{9}$$

It is clear which terms constitute vector-vector, axial-axial, or interference contributions. The dependence on the non-conserved part of the current is also explicit (note that $2mF_A = q^2 F_P$ is the condition for a conserved axial current).

3.2. *Isobar production*

We now consider quasi-free excitation of a nucleon resonance of spin J_R, parity π_R, and mass m_R. In the earlier treatment of electron scattering [1], the Bjorken-Walecka analysis [13] of the nucleon-isobar electromagnetic vertex was quite useful, since it provided a rather elegant expression for the nucleon tensor $T_{\mu\nu}$ in terms of the electromagnetic current helicity amplitudes for production of any isobar. Therefore, we extend this analysis to the nucleon-isobar weak interaction vertex. In this case, the vector-axial interference and the non-conservation of the weak current represent additional complications.

In analyzing the nucleon-isobar vertex, we work in the resonance rest frame and write $k' = (0, im_R)$, $q = (\boldsymbol{q}^*, iq_0^*)$ and (5c) and (5d) give

$$T_{\mu\nu} = T_1\delta_{\mu\nu} + T_2/m^2 k'_\mu k'_\nu + (T_\alpha + T_2 + 2T_\beta)/m^2 q_\mu q_\nu$$
$$+ (T_\beta + T_2)/m^2(k'_\mu q_\nu + k'_\nu q_\mu) + T_8/m^2\, \epsilon_{\mu\nu\sigma\tau}k'_\sigma q_\tau , \tag{10}$$

$$T_{\mu\nu} = m_R \epsilon_{q^*}\Omega^2 \sum_{M\lambda} \langle \pi_R J_R M | j_\mu^{(+)}(0) | \boldsymbol{q}^*\lambda\rangle\langle \boldsymbol{q}^*\lambda | j_\nu^{(-)}(0) | \pi_R J_R M\rangle . \tag{11}$$

The problem is now to use angular momentum conservation and the transformation properties of the current in a study of the matrix element for isobar excitation. We start by expanding the initial nucleon state in eigenstates of angular momentum and parity $|q\pi jm\rangle$; choosing the z-axis to be in the $\boldsymbol{q}^*$ direction we have [12]

$$|\boldsymbol{q}^*\lambda = +\tfrac{1}{2}\rangle = \sum_{jm} \left(\frac{2j+1}{8\pi}\right)^{\frac{1}{2}} \{|q+jm\rangle + |q-jm\rangle\} . \tag{12}$$

$$|\boldsymbol{q}^*\lambda = -\tfrac{1}{2}\rangle = \sum_{jm} \left(\frac{2j+1}{8\pi}\right)^{\frac{1}{2}} (-1)^{j-\frac{1}{2}} \{|q+jm\rangle - |q-jm\rangle\} .$$

The matrix elements in (11) are now taken between states of definite angular momentum and parity, and we can use the Wigner-Eckart theorem to extract the M-dependence of the matrix elements

$$\langle\pi_\mathrm{R} J_\mathrm{R} M|\boldsymbol{j}_\rho^{(\pm)}(0)|q^*\pi jm\rangle = (-1)^{J_\mathrm{R}-M} \begin{pmatrix} J_\mathrm{R} & 1 & j \\ -M & \rho & m \end{pmatrix} \langle\pi_\mathrm{R} J_\mathrm{R}||\boldsymbol{j}^{(\pm)}(0)||q^*\pi j\rangle ,$$

$$\langle\pi_\mathrm{R} J_\mathrm{R} M|j_0^{(\pm)}(0)|q^*\pi jm\rangle = (-1)^{J_\mathrm{R}-M} \begin{pmatrix} J_\mathrm{R} & 0 & j \\ -M & 0 & m \end{pmatrix} \langle\pi_\mathrm{R} J_\mathrm{R}||j_0^{(\pm)}(0)||q^*\pi j\rangle , \tag{13}$$

where the spherical components $\boldsymbol{j}_\rho$ are $\boldsymbol{j}_{\pm 1} = \mp 1/\sqrt{2}\,(j_x \pm ij_y)$ and $\boldsymbol{j}_0 = j_z$. We now introduce the following helicity amplitudes of the vector and axial currents:

$$f_\rho = \left(\frac{m_\mathrm{R}\epsilon_{q^*}\Omega^2}{8\pi m_\mathrm{R}^2}\right)^{\frac{1}{2}} \sum_j (2j+1)^{\frac{1}{2}} (-1)^{J_\mathrm{R}-\rho-\frac{1}{2}} \begin{pmatrix} J_\mathrm{R} & 1 & j \\ -\rho-\frac{1}{2} & \rho & \frac{1}{2} \end{pmatrix} \langle\pi_\mathrm{R} J_\mathrm{R}||\boldsymbol{V}(0)||q^*\pi j\rangle ,$$

$$g_\rho = \left(\frac{m_\mathrm{R}\epsilon_{q^*}\Omega^2}{8\pi m_\mathrm{R}^2}\right)^{\frac{1}{2}} \sum_j (2j+1)^{\frac{1}{2}} (-1)^{J_\mathrm{R}-\rho-\frac{1}{2}} \begin{pmatrix} J_\mathrm{R} & 1 & j \\ -\rho-\frac{1}{2} & \rho & \frac{1}{2} \end{pmatrix} \langle\pi_\mathrm{R} J_\mathrm{R}||\boldsymbol{A}(0)||q^*\pi j\rangle ,$$

$$f_\mathrm{c} = \left(\frac{m_\mathrm{R}\epsilon_{q^*}\Omega^2}{8\pi m_\mathrm{R}^2}\right)^{\frac{1}{2}} \langle\pi_\mathrm{R} J_\mathrm{R}||V_0(0)||q^*\pi j\rangle ,$$

$$g_\mathrm{c} = \left(\frac{m_\mathrm{R}\epsilon_{q^*}\Omega^2}{8\pi m_\mathrm{R}^2}\right)^{\frac{1}{2}} \langle\pi_\mathrm{R} J_\mathrm{R}||A_0(0)||q^*\pi j\rangle . \tag{14}$$

It is understood in (14) that only the appropriate values of the parity π contribute in the various amplitudes. Current conservation for the vector current

$$q_\mu\langle\pi_\mathrm{R} J_\mathrm{R} M|V_\mu^{(\pm)}(0)|\boldsymbol{q}^*\lambda\rangle = 0$$

implies that $q^* f_\mathrm{c} = q_0 f_\mathrm{c}$. This leads us to define the amplitude [14]

$$g_{\mathrm{d}} = \frac{q^*}{q^2}(q^* g_0 - q_0 g_{\mathrm{c}}) \ ,$$

which would vanish if the axial current were conserved. Finally we compute the T_j in terms of the helicity amplitudes. Inspection of (10) reveals that $T_{\mu\nu}$ has five independent form factors, and our approach is to evaluate five scalar quantities in terms of (10) and in terms of (11) to (14). A straightforward, although lengthy, calculation gives the results

$$\sum_{i=1}^{3} T_{ij} = 2m_{\mathrm{R}}^2 \{|f_+|^2 + |f_-|^2 + |f_0|^2 + |g_+|^2 + |g_-|^2 + |g_0|^2\}$$

$$= 3T_1 + |\boldsymbol{q}^*|^2/m^2 (T_2 + T_\alpha + 2T_\beta) \ ,$$

$$T_{44} = -2m_{\mathrm{R}}^2 \{|f_{\mathrm{c}}|^2 + |g_{\mathrm{c}}|^2\}$$

$$= T_1 - m_{\mathrm{R}}^2/m^2 T_2 - q_0^{*2}/m^2 (T_2 + T_\alpha + 2T_\beta)$$

$$- 2m_{\mathrm{R}} q_0^*/m^2 (T_2 + T_\beta) \ ,$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} q_i^* q_j^* T_{ij} = 2|\boldsymbol{q}^*|^2 m_{\mathrm{R}}^2 \{|f_0|^2 + |g_0|^2\}$$

$$= |\boldsymbol{q}^*|^2 \{T_1 + |\boldsymbol{q}^*|^2/m^2 (T_2 + T_\alpha + 2T_\beta)\} \ ,$$

$$\sum_{i=1}^{3} q_i^* (T_{i4} + T_{4i}) = 4im_{\mathrm{R}}^2 |\boldsymbol{q}^*| \mathrm{Re}\{f_0 f_{\mathrm{c}}^* + g_0 g_{\mathrm{c}}^*\}$$

$$= 2i|\boldsymbol{q}^*|^2 \{q_0^*/m^2 (T_2 + T_\alpha + 2T_\beta) + m_{\mathrm{R}}/m^2 (T_2 + T_\beta)\} \ ,$$

$$\sum_{i,j,k=1}^{3} \tfrac{1}{2}\epsilon_{ijk} T_{ij} q_k^* = -2im_{\mathrm{R}}^2 |\boldsymbol{q}^*| \mathrm{Re}\{f_- g_-^* - f_+ g_+^*\}$$

$$= -i|\boldsymbol{q}^*|^2 m_{\mathrm{R}}/m^2 T_8 \ . \tag{15}$$

This set of equations determines the T_j:

$$T_1 = m_{\mathrm{R}}^2 \{|f_+|^2 + f_-|^2 + |g_+|^2 + |g_-|^2\} \ , \tag{16a}$$

$$T_2 = q^2 m^2/|\boldsymbol{q}^*|^2 \{|f_+|^2 + |f_-|^2 + |g_+|^2 + |g_-|^2$$

$$+ 2q^2/|\boldsymbol{q}^*|^2 [\,|f_{\mathrm{c}}|^2 + |g_{\mathrm{c}}|^2 - 2q_0^*/|\boldsymbol{q}^*| \,\mathrm{Re}\,(g_{\mathrm{c}} g_{\mathrm{d}}^*) + q_0^{*2}/|\boldsymbol{q}^*|^2 |g_{\mathrm{d}}|^2]\} \ , \tag{16b}$$

$$T_\alpha = -m^2/q^2\,T_1 + \left(\frac{k\cdot q}{q^2}\right)^2 T_2 + 2m^2|g_d|^2 q^4/|\boldsymbol{q}^*|^6 \left\{(m_R + q_0^*)^2 - q_0^{*2}\left(\frac{k\cdot q}{q^2}\right)\right\} - 4m^2 m_R (k\cdot q)/|\boldsymbol{q}^*|^3\,\mathrm{Re}(g_c g_d^*)\,, \tag{16c}$$

$$T_\beta = -k\cdot q/q^2\,T_2 - 2m^2 m_R q_0 q^2/|\boldsymbol{q}^*|^4\,|g_d|^2 + 2m^2 m_R q^2/|\boldsymbol{q}^*|^3\,\mathrm{Re}(g_c g_d^*)\,, \tag{16d}$$

$$T_8 = 2m^2 m_R/|\boldsymbol{q}^*|\,\mathrm{Re}\{f_- g_-^* - f_+ g_+^*\}\,, \tag{16e}$$

where

$$q_0^* = (q^2 + m^2 - m_R^2)/(2m_R)\,,$$

$$|\boldsymbol{q}^*|^2 = (q^2 + m^2 + m_R^2)/(4m_R^2) - m^2\,,$$

$$k\cdot q = \tfrac{1}{2}(q^2 + m_R^2 - m^2)\,.$$

Again the vector-vector, axial-axial, and interference terms are easily identified, and the non-conserved part of the axial current appears in terms containing g_d. The results for T_1 and T_2 agree with the Bjorken-Walecka results if all g's are set to zero. Eq. (16) is the main result of this section. It expresses the T_j needed in our general expression for quasi-free processe (7) directly in terms of the helicity amplitudes for weak excitation of any isobar, and these are in turn given directly by the matrix elements of the vector and axial currents between nucleon and isobar states. Finally, we note that in the extreme relativistic limit ($m_\ell = 0$, $\chi = \theta$) the free nucleon cross section is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{lab}} = \frac{G^2}{2\pi^2}\,\frac{\epsilon_2^2\cos^2\left(\frac{\theta}{2}\right)}{\left(1+\frac{2\epsilon_1}{m}\sin^2\frac{1}{2}\theta\right)}\,\Big\{q^4/|\boldsymbol{q}^*|^4\left[|f_c|^2 + |g_c|^2\right]$$

$$+ \left(q^2/(2|\boldsymbol{q}^*|^2) + \frac{m_R^2}{m^2}\tan^2(\tfrac{1}{2}\theta)\right)\cdot\left[|f_+|^2 + |f_-|^2 + |g_+|^2 + |g_-|^2\right]$$

$$+ q_0^{*2} q^4/|\boldsymbol{q}^*|^6\,|g_d|^2 - 2q_0^* q^4/|\boldsymbol{q}^*|^5\,\mathrm{Re}(g_c g_d^*)$$

$$- 2\sin\tfrac{1}{2}\theta\,\frac{(q^2\cos^2\frac{1}{2}\theta + |\boldsymbol{q}|^2\sin^2\frac{1}{2}\theta)^{\frac{1}{2}}}{|\boldsymbol{q}^*|\cos^2\frac{1}{2}\theta}\,\mathrm{Re}\left[f_- g_-^* - f_+ g_+^*\right]\Big\}\,.$$

4 NUMERICAL RESULTS

We now present some results for quasi-elastic scattering and for quasi-free production of the (3 - 3) resonance, our assumption being that the latter process dominates single-pion production in the appropriate region of lepton energy loss. We simply replace the nuclear form factors W_j in (3) by their expressions (7) in terms of the a_j and T_j. The a_j contain the nuclear physics, and we shall use the analytic results of the appendix for the simple Fermi gas model. Finally, the T_j are given in terms of the "elementary" nucleon form factors in (9) and (16) for quasi elastic scattering and isobar excitation, respectively. The form factors F_1 and F_2 are obtained from the electromagnetic form factors by CVC, and the second class currents F_S and F_T are assumed to be absent. We use $F_A(q^2) = F_A(0)/(1+q^2/(0.75\,\mathrm{GeV})^2)^2$ and take $F_P(q^2) = 2mF_A(q^2)/(q^2+m_\pi^2)$. This prescription for F_P should be good for small q^2, but the behaviour at large q^2 is not known. We have used Zucker's helicity amplitudes [14] for isobar production. He has performed a fully relativistic N/D calculation of the weak production process in essentially the same model used by Pritchett, Walecka and Zucker [15] in a study of resonance electroproduction. This earlier calculation sets the normalization for Zucker's results [14]. In addition, we have folded in a reasonable width [16] for the N*(1236). The values of k_F and ϵ_1 (see appendix) have been taken from fits of the same model [1] to electron scattering ‡, while

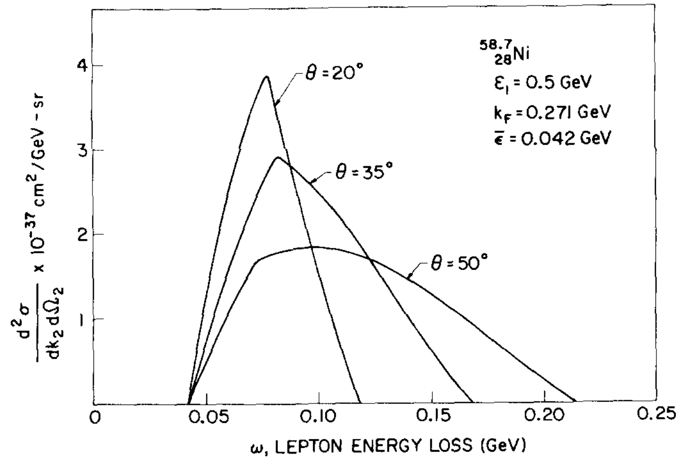


Fig. 2. The cross section for quasi-elastic neutrino scattering on $^{58.7}$Ni at low momentum transfer ($|\boldsymbol{q}| \sim k_F$) showing the effect of the exclusion principle. Note: $\bar{\epsilon} = \epsilon_1$.

‡ The neutron and proton Fermi momenta were taken as $k_{F_n} = (2N/A)^{1/3}k_F$ and $k_{F_p} = (2Z/A)^{1/2}k_F$, respectively. The implication here is that, for a given k_F, the density of nuclear matter is independent of the ratio of neutrons to protons. This assumption is supported by elastic electron-scattering data, which show that nuclear half-density radii vary as $A^{1/3}$ and that $A\rho_0/Z$, where ρ_0 is the central proton density, is roughly constant for heavy nuclei.

the interaction energy of the recoil particle is chosen to be zero (as was done in ref. [1]). Finally, all cross sections have been computed for a monochromatic ν_μ beam; averaging over a real neutrino spectrum could be easily done using the analytic results of the appendix.

In fig. 2 we show the cross section for quasi-elastic neutrino scattering on $^{58.7}$Ni at an incident energy of 0.5 GeV and scattering angles of 20°, 35°

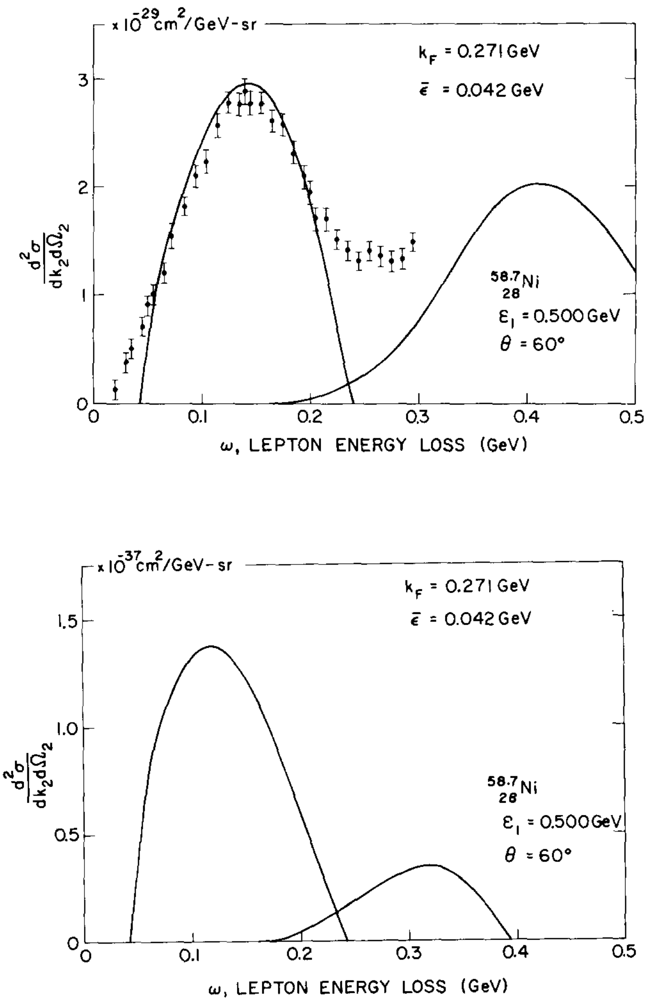


Fig. 3. Quasi-elastic and N*(1236) production cross sections on $^{58.7}$Ni for (a) electron scattering and (b) neutrino scattering at 0.5 GeV incident energy and a 60° scattering angle.

and 50°. The dominant feature in this region where the momentum transfer $|\boldsymbol{q}|$ is comparable to the Fermi momentum is the effect of the exclusion principle. This reduction in the number of "available" final states is directly responsible for the straight-line portion on the left-hand side of each curve. The peaks are offset from the origin by an energy corresponding to the binding energy of the target nucleon. As the scattering angle increases the momentum transfer $|\boldsymbol{q}|$ increases and the exclusion effect becomes diminished. Bell and Llewellyn-Smith [6] find the shell model is "less exclusive" than the Fermi gas model.

The quasi-elastic and $N^*(1236)$ peaks in fig. 3a are for electron scattering on $^{58.7}$Ni at incident energy 0.5 GeV and scattering angle 60° plotted with the experimental data [7]. The corresponding peaks in fig. 3b are for neutrino scattering at the same angle and incident energy. The difference in endpoint for the N^* peaks is due to the additional energy required for the muon rest mass. In this region of momentum transfer the two peaks are well separated and exclusion effects are minimal. With incident beams of moderate energy resolution, the neutron (proton) Fermi momentum could be determined directly from the width of the quasi-elastic peak for incoming neutrinos (antineutrinos). It would then be interesting to compare $k_{\mathrm{F}_{\mathrm{p}}}$ measured this way with the value obtained from electron scattering [7].

At NAL energies, one would like to extract the neutrino cross section from the quasi-elastic peak. However, we point out that the widths of the quasi-elastic and isobar peaks are proportional not only to the Fermi momentum but also to the momentum transfer $|\boldsymbol{q}|$. Consequently, we can expect these peaks to overlap appreciably for high energies at all but the smallest scattering angles. This is seen explicitly in fig. 4, where the results are shown for 15 GeV incident neutrinos and a scattering angle of 5°. The N^* width has been folded in. It is clear that one must be quite careful in extracting single nucleon data in such a situation.

The effect of the exclusion principle and nuclear Fermi motion may be easily seen by examining the ratio

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}q^2}\right)_{\text{nuclear}} \Big/ \left(\frac{\mathrm{d}\sigma}{\mathrm{d}q^2}\right)_{\text{nucleon}}$$

obtained by integrating $\mathrm{d}^2\sigma/\mathrm{d}q^2\mathrm{d}\omega$. For stationary non-identical nucleons this would simply be the number of targets in the nucleus. In fig. 5 we plot the sum rule corresponding to this ratio which was derived by Berman [17] for a non-relativistic Fermi gas and is discussed by Walecka [18]. In the same figure we show the ratio computed for ^{208}Pb at 1 GeV incident neutrino energy. This curve is smaller than the sum rule primarily because of the binding energy of the struck nucleon. The cross section $\mathrm{d}^2\sigma/\mathrm{d}q^2\mathrm{d}\omega$ which we integrate over ω contains as a recoil factor the ratio of the energies of the observed and incident leptons. The energy required to remove the recoiling nucleon from the nuclear potential necessarily reduces the energy available to the observed lepton. At higher incident energies this effect becomes negligible. The sharp falloff of the cross section ratio at the high q^2 side of the graph is due to the fact that not all energy transfers are

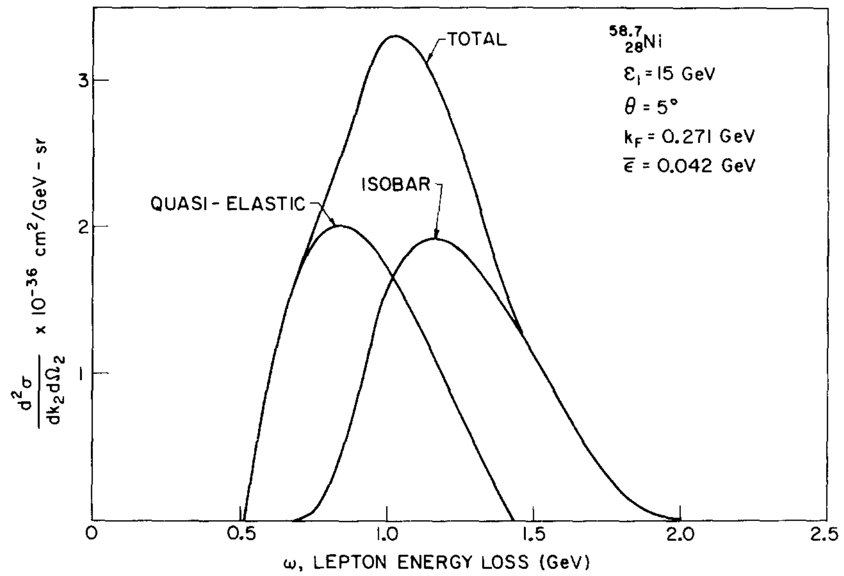


Fig. 4. Quasi-elastic and $N^*(1236)$ production cross sections and their sum for ^{208}Pb at an incident energy of 15 GeV and a scattering angle of 5^o.

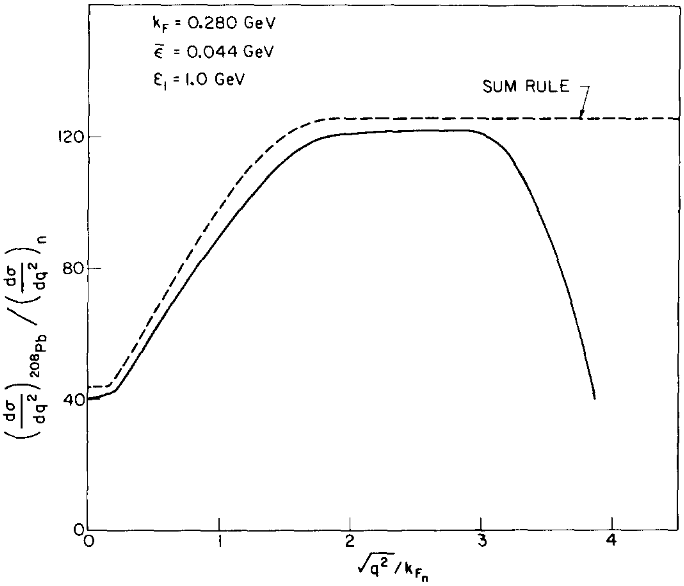


Fig. 5. The calculated ratio $(d\sigma/dq^2)_{208_{Pb}}/(d\sigma/dq^2)_n$ at incident neutrino energy 1 GeV and a non-relativistic sum rule.

possible (due to energy conservation) at fixed q^2. The sum rule integrates over all energy losses; the experiment must conserve energy.

5. CONCLUDING REMARKS

In conclusion we make some brief comments on the validity of our model. First, the inclusion of a binding energy in the nucleon single particle energies (see appendix) violates strict conservation of the vector current. However, we could "fix this up" by adding a term $-iF_1\epsilon_1 q_\mu/q^2$ to the nuclear matrix element $\langle k'|V_\mu|k\rangle$, but we readily see that this contributes to the cross section only to order m_ℓ^2. Second, our use of plane waves for the ejected nucleon states is justified when only the final lepton is observed. The reason is simply that we can rewrite the nuclear response function so that the final state is eliminated through closure:

$$W_{\mu\nu} = (2\pi)^3\Omega \,\overline{\sum_i}\sum_f \,\delta^{(4)}(q+p'-p)\langle p|j_\nu^{(-)}(0)|p'\rangle\langle p'|j_\mu^{(+)}(0)|p\rangle E\ ,$$

$$= (2\pi)^3\Omega \,\overline{\sum_i} \int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x}\langle p|j_\nu^{(-)}(x)j_\mu^{(+)}(0)|p\rangle E\ . \qquad (18)$$

Now the final states $|p'\rangle$ which are "reached", but not observed, in the (ν, ℓ) or (ℓ, ℓ') processes are extremely complicated; for example, they can contain more than one particle in the continuum plus excitation of the residual nucleus. Nevertheless it is clear from (18) that we need only a complete set of A-particle states at the appropriate energy. It is incorrect to add single-particle matrix elements between single nucleon initial states and final states including a nuclear absorption factor. We have simply constructed the A-body nuclear states out of plane waves. Again the binding energy introduces a complication: we are using a complete set of states with slightly incorrect energies. However, for large momentum transfers and energy losses, the nucleons are "almost free" and the error should be very small.

Consider next our description of the initial target state. The important quantity in determining the quasi-free reaction is the one-body momentum distribution. The a_j can be computed numerically from (7) for any $n(k)$, but we have used the analytic results for $n(k) = \theta(k_\mathrm{F} - k)$. We expect this to give good results for two reasons: the very high momentum components arising from short range correlations have been shown to modify the (e, e') quasi-elastic cross section only slightly [19], because the high momentum tail is very small in magnitude; second, the quasi-elastic cross section, for large momentum transfer $|\boldsymbol{q}| \gtrsim 2k_\mathrm{F}$ is insensitive to the detailed shape of $n(k)$ near the Fermi surface. This can be seen explicitly by comparing the Fermi gas calculation of (e, e') quasi-elastic scattering [1] with fig. 15 of Donnelly's calculation [20].

Finally, we summarize our main points. We have computed the cross section for neutrino quasi-elastic scattering and for quasi-free isobar excitation, retaining relativistic kinematics for the recoiling particle and the full relativistic hadronic weak vertex. This is the first calculation of the $N^*(1236)$ and should be useful in separating the strictly nuclear effects from pion production. The isobar cross section is expressed in terms of the helicity amplitudes of the weak current, obtained from a general analysis of the nucleon-isobar vertex. For the simple Fermi gas model, we can evaluate the cross section for fixed incident neutrino energy analytically, allowing us to average over any neutrino spectrum easily. We stress that exactly the same model has already been applied very succesfully to inelastic electron scattering from complex nuclei, suggesting that our calculation should reliably predict the dominant features of the nuclear weak response function.

We would like to thank Professor J. D. Walecka for suggesting this calculation and for many helpful discussions while the work was in progress. We also thank Dr. P. A. Zucker for sending us a table of his isobar helicity amplitudes.

APPENDIX

In this appendix we give the analytic expressions for the a_j defined in (5b) and (7) for the case of a simple Fermi gas $n_\alpha(k) = \theta(k_{F_\alpha} - k)$. To evaluate the a_j we assume $\epsilon_k = (|\boldsymbol{k}|^2 + m^2)^{1/2} - \epsilon_1$, $\epsilon_{\boldsymbol{k}-\boldsymbol{q}} = (|\boldsymbol{k}-\boldsymbol{q}|^2 + m'^2)^{1/2} - \epsilon_2$ which allow for different binding energies of initial and final particles. For quasi-elastic scattering, $m' = m$; for resonance production, $m' = m_R$.

We define

$$\epsilon = (|\boldsymbol{k}|^2 + m^2)^{\frac{1}{2}} ; \qquad \omega_{\text{eff}} = \omega + \epsilon_2 - \epsilon_1 ;$$

$$q^2_{\text{eff}} = |\boldsymbol{q}|^2 - \omega^2_{\text{eff}} + m'^2 - m^2 ; \qquad a = \epsilon_1 \left(1 + \frac{\epsilon_2}{\omega}\right) ;$$

$$b = \epsilon_2 \left(1 - \frac{\epsilon_1}{\omega}\right) ; \qquad c = -\omega_{\text{eff}}/|\boldsymbol{q}| ;$$

$$d = q^2_{\text{eff}}/(2|\boldsymbol{q}|m) .$$

We introduce

$$b_j = \int \mathrm{d}\boldsymbol{k} f(\boldsymbol{k}, \boldsymbol{q}, \omega)\left(\frac{\epsilon}{m}\right)^j , \tag{A1'}$$

$$b_0 = \frac{m_T \Omega}{(2\pi)^2 |\boldsymbol{q}|} \{\epsilon + a \ln(\epsilon - \epsilon_1) + b \ln(\epsilon - \epsilon_1 + \omega)\}\Big|^{\epsilon_u}_{\epsilon_l} , \tag{A1''}$$

$$b_1 = \frac{m_T \Omega}{(2\pi)^2 |\boldsymbol{q}|} \frac{1}{m} \{\tfrac{1}{2}\epsilon^2 + a(\epsilon + \epsilon_1 \ln(\epsilon - \epsilon_1)) + b(\epsilon + (\epsilon_1 - \omega)\ln(\epsilon - \epsilon_1 + \omega))\}\Big|^{\epsilon_u}_{\epsilon_l} , \tag{A1'''}$$

$$b_2 = \frac{m_T \Omega}{(2\pi)^2 |\boldsymbol{q}|} \frac{1}{m^2} \left\{ \tfrac{1}{3}\epsilon^3 + a\left(\frac{\epsilon^2}{2} + \epsilon_1 \epsilon + \epsilon_1^2 \ln(\epsilon - \epsilon_1)\right) + b\left(\tfrac{1}{2}\epsilon^2 + (\epsilon_1 - \omega)\epsilon + (\epsilon_1 - \omega)^2 \ln(\epsilon - \epsilon_1 + \omega)\right)\right\}\Big|_{\epsilon_l}^{\epsilon_u} . \quad (\mathrm{A1''''})$$

The integration limits ϵ_u and ϵ_l are determined by the momentum distributions and the delta function. The delta function determines that $|\boldsymbol{k}| \cos\tau = c\epsilon + dm$. The cross section will vanish unless $|\cos\tau| \leq 1$ which implies that $\epsilon_l \geq m(cd + \sqrt{1 - c^2 + d^2})/(1 - c^2)$. For quasi-elastic scattering only, the exclusion principle for the final state means $\epsilon_l \geq (k_{F_f}^2 + m^2)^{1/2} - \omega_{\mathrm{eff}}$ where k_{F_f} is the Fermi momentum associated with the outgoing nucleon. The upper limit ϵ_u is simply determined by the highest level in the target, i.e. $\epsilon_u = (k_{F_i}^2 + m^2)^{1/2}$. We then take for ϵ_l the largest value imposed by the above constraints. Of course if $\epsilon_l > \epsilon_u$ the cross section is zero. Finally, in terms of the b_j, we have

$$a_1 = b_0 , \qquad a_2 = b_2 - b_0 , \qquad a_3 = c^2 b_2 + 2cdb_1 + d^2 b_0 ,$$

$$a_4 = b_2 - \frac{2\epsilon_1}{m} b_1 + \frac{\epsilon_1^2}{m^2} b_0 , \qquad a_5 = cb_2 + \left(d - \frac{\epsilon_1 c}{m}\right) b_1 - \frac{\epsilon_1 d}{m} b_0 ,$$

$$a_6 = cb_1 + db_0 , \qquad a_7 = b_1 - \frac{\epsilon_1}{m} b_0 . \qquad (\mathrm{A2})$$

For the Fermi gas, Ω may be replaced by $3\pi^2 N/(k_{F_n}^3)$, where N is the number of neutrons in the nucleus. The binding energies of the target nucleons are chosen so that neutrons and protons at the Fermi surfaces will have the same binding energy.

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