Test of Lorentz and CPT violation with neutrinos

Outline
1. Why Lorentz violation is interesting with short baseline neutrino oscillations?
2. Test of Lorentz violation with neutrino oscillations
3. Lorentz violation with LSND
4. Lorentz violation with MiniBooNE neutrino data
5. Lorentz violation with MiniBooNE anti-neutrino data
6. Conclusion
1. Why Lorentz violation is interesting with short baseline neutrino oscillations?

2. Test of Lorentz violation with neutrino oscillations

3. Lorentz violation with LSND

4. Lorentz violation with MiniBooNE neutrino data

5. Lorentz violation with MiniBooNE anti-neutrino data

6. Conclusion
1. Why Lorentz violation is interesting in short baseline neutrino oscillations?

Model independent neutrino oscillation data is the function of neutrino energy and baseline
- Addition of Lorentz violation offers rich energy dependence on oscillation length
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See Raghavan’s talk
1. Why Lorentz violation is interesting in short baseline neutrino oscillations?

Model independent neutrino oscillation data is the function of neutrino energy and baseline
- Addition of Lorentz violation offers rich energy dependence on oscillation length

How about model like this?
→ it looks no problem!

It is very interesting to test Lorentz violation to short baseline neutrino oscillation experiments!
1. Why Lorentz violation is interesting with short baseline neutrino oscillations?

2. Test of Lorentz violation with neutrino oscillations

3. Lorentz violation with LSND

4. Lorentz violation with MiniBooNE neutrino data

5. Lorentz violation with MiniBooNE anti-neutrino data

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2. Test of Lorentz violation with neutrino oscillations

How to detect Lorentz violation?

Lorentz violation is realized as a coupling of particle fields and the background fields, so the basic strategy is to find the Lorentz violation is;

(1) choose the coordinate system to compare the experimental result
(2) write down Lagrangian including Lorentz violating terms under the formalism
(3) write down the observables using this Lagrangian

Scientific American (Sept. 2004)
2. Test of Lorentz violation with neutrino oscillations

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The standard choice of the coordinate is Sun-centred celestial equatorial coordinates
2. Test of Lorentz violation with neutrino oscillations

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As a standard formalism for the general search of Lorentz violation, Standard Model Extension (SME) is widely used in the community. SME is self-consistent low-energy effective theory with Lorentz and CPT violation within conventional QM (minimum extension of QFT with Particle Lorentz violation)

Modified Dirac Equation (MDE) for neutrinos

\[ i(\Gamma^\nu_{AB}\delta_{\nu} - M_{AB})\nu_B = 0 \]

SME parameters

\[
\Gamma^\nu_{AB} = \gamma^\nu \delta_{AB} + c^\mu_{AB} \gamma_\mu + d^\mu_{AB} \gamma_\mu \gamma_5 + e^\nu_{AB} + i f^\nu_{AB} \gamma_5 + \frac{1}{2} g^\lambda_{AB} \sigma^{\lambda\mu} \\
M_{AB} = m_{AB} + im_{5AB} \gamma_5 + a^\mu_{AB} \gamma_\mu + b^\mu_{AB} \gamma_5 \gamma_\mu + \frac{1}{2} H^\mu_{AB} \sigma_{\mu\nu}
\]
How to detect Lorentz violation?

Lorentz violation is realized as a coupling of particle fields and the background fields, so the basic strategy is to find the Lorentz violation is;

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The observables can be, energy spectrum, frequency of atomic transition, neutrino oscillation probability, etc. Among the non standard phenomena predicted by Lorentz violation, the smoking gun is the sidereal time dependence of the observables.

ex) Sidereal variation of MiniBooNE signal

\[
P_{\nu_e \rightarrow \nu_\mu} \sim \frac{|(h_{\text{eff}})_{e\mu}|^2 L^2}{(hc)^2}
\]

\[
= \left( \frac{L}{hc} \right)^2 |(C)_{e\mu} + (A_s)_{e\mu} \sin w_\oplus T_\oplus + (A_c)_{e\mu} \cos w_\oplus T_\oplus + (B_s)_{e\mu} \sin 2w_\oplus T_\oplus + (B_c)_{e\mu} \cos 2w_\oplus T_\oplus |^2
\]

Sidereal frequency \( w_\oplus = \frac{2\pi}{23h56m4.1s} \)

Sidereal time \( T_\oplus \)

Sidereal variation analysis for MiniBooNE is 5 parameter fitting problem
1. Why Lorentz violation is interesting with short baseline neutrino oscillations?

2. Test of Lorentz violation with neutrino oscillations

3. Lorentz violation with LSND

4. Lorentz violation with MiniBooNE neutrino data

5. Lorentz violation with MiniBooNE anti-neutrino data

6. Conclusion
3. Lorentz violation with LSND

Neutrino mode low energy excess
LSND experiment at Los Alamos observed excess of anti-electron neutrino events in the anti-muon neutrino beam.

\[ 87.9 \pm 22.4 \pm 6.0 \ (3.8 \sigma) \]

This is not predicted by neutrino standard model (νSM), so it is interesting to test Lorentz violation with LSND data.
3. Lorentz violation with LSND

Data taking period
- if data taking is uniform with time, all day-night effect would be smeared out (not the case for LSND)

Solar time distribution
- to check day-night effect

Flatness test
- to test the consistency with no sidereal variation

Flat hypothesis (solar time)
P(K-S)=0.39

Flat hypothesis (sidereal time)
P(K-S)=0.23

Neutrino mode excess is compatible with flat hypothesis.
3. Lorentz violation with LSND

Unbinned likelihood fit
- maximum statistics power for low statistics data (186 events).

3 parameter fit result
- statistics doesn’t allow to fit 5 parameters simultaneously.

$$P_{\nu_e \to \nu_\mu} = \left( \frac{L}{\hbar c} \right)^2 \left| (C)e_{\mu} + (A_s)e_{\mu} \sin \omega \tau_\odot + (A_c)e_{\mu} \cos \omega \tau_\odot \right|^2$$
- because of the nature of the formula, solution is duplicated.

2 distinct solutions in 1-σ region (unit $10^{-19}$ GeV)
- solution 1: this solution include maximum loglikelihood (MLL) point, and sidereal time dependent solution.
- solution 2: this solution doesn’t include MLL point, and sidereal time independent solution.

<table>
<thead>
<tr>
<th>soln1</th>
<th>(\delta_1)</th>
<th>soln2</th>
<th>(\delta_1)</th>
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<tbody>
<tr>
<td>((C)e_{\mu})</td>
<td>±0.2</td>
<td>1.0</td>
<td>±3.3</td>
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<tr>
<td>((A_s)e_{\mu})</td>
<td>±4.0</td>
<td>1.4</td>
<td>±0.1</td>
</tr>
<tr>
<td>((A_c)e_{\mu})</td>
<td>±1.9</td>
<td>1.8</td>
<td>±0.5</td>
</tr>
</tbody>
</table>

SME parameter

- [Equation for SME parameters]

LSND Collaboration,
PRD72(2005)076004
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4. Lorentz violation with MiniBooNE neutrino data

Neutrino mode low energy excess

MiniBooNE didn't see the signal at the region where LSND data suggested under the assumption of standard 2 massive neutrino oscillation model, but MiniBooNE did see the excess where neutrino standard model doesn't predict the signal.

The energy dependence of MiniBooNE is reproducible by Lorentz violation motivated model, such as Puma model (next talk).

The low energy excess may have sidereal time dependence.

All backgrounds are measured in other data sample and their errors are constrained.
4. Lorentz violation with MiniBooNE neutrino data

Proton on target day-night variation

Since beam is running almost all year, any solar time structure, mainly POT day-night variation, is washed out in sidereal time.

Time dependent systematic errors are evaluated through observed CCQE events. The dominant source is POT variation.

POT makes 6% variation, but including this gives negligible effect in sidereal time distribution.

Therefore later we ignore all time dependent systematic errors.
4. Lorentz violation with MiniBooNE neutrino data

Flatness test

The flatness hypothesis is tested in 2 ways, Pearson’s $\chi^2$ test ($\chi^2$ test) and unbinned Kolmogorov-Smirnov test (K-S test).

Flat hypothesis (solar time)
$P(K-S)=0.64$

Flat hypothesis (sidereal time)
$P(K-S)=0.14$

Neutrino mode excess is compatible with flat hypothesis.
4. Lorentz violation with MiniBooNE neutrino data

\[ P_{\nu_e \to \nu_\mu} = \left( \frac{L}{\hbar c} \right)^2 \left( (C)_{\bar{e}\bar{\mu}} + (A_s)_{\bar{e}\bar{\mu}} \sin w_{\oplus} T_{\oplus} + (A_c)_{\bar{e}\bar{\mu}} \cos w_{\oplus} T_{\oplus} \right)^2 \]

Unbinned loglikelihood method

This method utilizes the highest statistical power

Flat hypothesis (solar time)
\[ P(K-S)=0.64 \]

Flat hypothesis (sidereal time)
\[ P(K-S)=0.14 \]

After fit (sidereal time)
\[ P(K-S)=0.98 \]

C-parameter is statistically significant value, but this is sidereal independent parameter.

Solution discovered by fit improve goodness-of-fit, but flat hypothesis is already a good solution.
4. Lorentz violation with MiniBooNE neutrino data

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Solution discovered by fit improve goodness-of-fit, but flat hypothesis is already a good solution.

For neutrino mode, P(K-S)=14% before fit, so data is consistent with no sidereal variation hypothesis. After fit, P(K-S)=98%, however, the best fit point has strong signal on C-term (not sidereal time dependent).
1. Why Lorentz violation is interesting with short baseline neutrino oscillations?

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Anti-neutrino mode low energy excess
MiniBooNE did see the signal at the region where LSND data suggested under the assumption of standard two massive neutrino oscillation model.

If the excess were Lorentz violation, the excess may have sidereal time dependence.
5. Lorentz violation with MiniBooNE anti-neutrino data

Flatness test

The flatness hypothesis is tested in 2 ways, Pearson’s $\chi^2$ test ($\chi^2$ test) and unbinned Kolmogorov-Smirnov test (K-S test).

Flat hypothesis (solar time)
P(K-S)=0.69

Flat hypothesis (sidereal time)
P(K-S)=0.08

Neutrino mode excess is compatible with flat hypothesis.
5. Lorentz violation with MiniBooNE anti-neutrino data

Unbinned loglikelihood method

$$P_{\nu_e \rightarrow \nu_\mu} = \left( \frac{L}{hC} \right)^2 \left| (C)_{e\mu} + (A_s)_{e\mu} \sin w_T + (A_c)_{e\mu} \cos w_T \right|^2$$

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Flat hypothesis (solar time)
P(K-S)=0.69

Flat hypothesis (sidereal time)
P(K-S)=0.08

After fit (sidereal time)
P(K-S)=0.63

Large As- and Ac- terms are preferred within 1-$\sigma$
(sidereal time dependent solution).

2-$\sigma$ contour encloses large C-term (sidereal time independent solution).
5. Lorentz violation with MiniBooNE anti-neutrino data

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Flat hypothesis (solar time)
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2-σ contour encloses large C-term (sidereal time independent solution).

For anti-neutrino mode, P(K-S)=8% before fit, so data is consistent with no sidereal variation hypothesis. After fit, P(K-S)=63%, also, the best fit point has signal on As- and Ac-term (sidereal time dependent)
5. Lorentz violation with MiniBooNE anti-neutrino data

Unbinned loglikelihood method

This method utilizes the highest statistical power

Flat hypothesis (solar time)
\[ P(K-S)=0.69 \]

Flat hypothesis (sidereal time)
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After fit (sidereal time)
\[ P(K-S)=0.63 \]

Large As- and Ac- terms are preferred within 1-\(\sigma\) (sidereal time dependent solution).

2-\(\sigma\) contour encloses large C-term (sidereal time independent solution).

For anti-neutrino mode, \(P(K-S)=8\%\) before fit, so data is consistent with no sidereal variation hypothesis. After fit, \(P(K-S)=63\%\), also, the best fit point has signal on As- and Ac-term (sidereal time dependent).

Fake data \(\Delta\chi^2\) study says there is 3\% chance this signal is by random fluctuation.

Fake data distribution (without signal) overlaid on data with 1-\(\sigma\) volume

Fake data distribution (with signal) overlaid on data with 1-\(\sigma\) volume
6. Conclusions

Lorentz and CPT violation has been shown to occur in Planck scale physics.

LSND and MiniBooNE data suggest Lorentz violation is an interesting solution of neutrino oscillation.

MiniBooNE neutrino mode summary
P(K-S)=14% before fit, so data is consistent with no sidereal variation hypothesis. After fit, P(K-S)=98%, however, the best fit point has strong signal on C-term (not sidereal time dependent).

MiniBooNE anti-neutrino mode summary
P(K-S)=8% before fit, so data is consistent with no sidereal variation hypothesis. After fit, P(K-S)=63%, also, the best fit point has strong signal on As- and Ac-term (sidereal time dependent).

Extraction of SME coefficients is undergoing.
Thank you for your attention!
Backup
2. Test of Lorentz violation with neutrino oscillations

Sidereal variation of neutrino oscillation probability for MiniBooNE (5 parameters)

\[
P_{\nu_e \to \nu_\mu} = \left( \frac{L}{\hbar c} \right)^2 \left[ (C)_{\theta \mu} + (A_s)_{\theta \mu} \sin \theta \Theta + (A_c)_{\theta \mu} \cos \theta \Theta + (B_s)_{\theta \mu} \sin 2\theta \Theta + (B_c)_{\theta \mu} \cos 2\theta \Theta \right]^2
\]

Expression of 5 observables (14 SME parameters)

\[
(C)_{\theta \mu} = (a_L)_{\theta \mu}^T - N^2 (a_L)_{\theta \mu}^Z + E \left[ -\frac{1}{2} (3 - N^2 N^2) (c_L)_{\theta \mu}^{TT} + 2 N^2 (c_L)_{\theta \mu}^{TZ} + \frac{1}{2} (1 - 3 N^2 N^2) (c_L)_{\theta \mu}^{ZZ} \right]
\]

\[
(A_s)_{\theta \mu} = N^Y (a_L)_{\theta \mu}^X - N^X (a_L)_{\theta \mu}^Y + E \left[ -2 N^Y (c_L)_{\theta \mu}^{TX} + 2 N^Y (c_L)_{\theta \mu}^{TY} + 2 N^X N^Z (c_L)_{\theta \mu}^{XZ} - 2 N^X N^Z (c_L)_{\theta \mu}^{YZ} \right]
\]

\[
(A_c)_{\theta \mu} = -N^X (a_L)_{\theta \mu}^X - N^Y (a_L)_{\theta \mu}^Y + E \left[ 2 N^X (c_L)_{\theta \mu}^{TX} + 2 N^Y (c_L)_{\theta \mu}^{TY} - 2 N^X N^Z (c_L)_{\theta \mu}^{XZ} - 2 N^X N^Z (c_L)_{\theta \mu}^{YZ} \right]
\]

\[
(B_s)_{\theta \mu} = E \left[ N^X N^Y (c_L)_{\theta \mu}^{XX} - (c_L)_{\theta \mu}^{YY} \right] - (N^X N^Y - N^Y N^Y) (c_L)_{\theta \mu}^{XY}
\]

\[
(B_c)_{\theta \mu} = E \left[ -\frac{1}{2} (N^X N^X - N^Y N^Y) (c_L)_{\theta \mu}^{XX} - (c_L)_{\theta \mu}^{YY} \right] - 2 N^X N^Y (c_L)_{\theta \mu}^{XY}
\]

\[
\begin{pmatrix}
N^X \\
N^Y \\
N^Z
\end{pmatrix} =
\begin{pmatrix}
\cos \chi \sin \theta \cos \phi - \sin \chi \cos \theta \\
\sin \theta \sin \phi \\
-\sin \chi \sin \theta \cos \phi - \cos \chi \cos \theta
\end{pmatrix}
\]

coordinate dependent direction vector
(depending on the latitude of FNAL, location of BNB and MiniBooNE detector)
5. Lorentz violation with MiniBooNE neutrino data

Unbinned extended maximum likelihood fit

- It has the maximum statistic power
- Assuming low energy excess is Lorentz violation, extract Lorentz violation parameters (SME parameters) from unbinned likelihood fit.

\[
\Lambda = \frac{e^{-(\mu_s + \mu_b^v)}}{N!} \prod_{i=1}^{N} (\mu_s F_s^i + \mu_b^v F_b^i) \times \frac{1}{\sqrt{2\pi \sigma_b^2}} \exp \left( -\frac{(\mu_b^v - \mu_b)^2}{2\sigma_b^2} \right)
\]

\( N \) total number of event
\( \mu_s \) predicted signal event number, function of fitting parameters
\( \mu_b \) predicted background event number
\( F_s \) probability distribution of signal, function of sidereal time and fitting parameters
\( F_b \) probability distribution of background, not function of sidereal time
\( \sigma_b \) the \( 1 - \sigma \) error of predicted the background
\( \mu_b^v \) floating background event number floating within \( 1 - \sigma \)
5. Lorentz violation with MiniBooNE neutrino data

Time distribution of MiniBooNE neutrino mode low energy region

MiniBooNE data taking is reasonably uniform, so all day-night effect is likely to be washed out in sidereal time distribution.

solar local time
24h00m00s (86400s)
sidereal time
23h56m04s (86164s)
5. Lorentz violation with MiniBooNE neutrino data

Null hypothesis test

The flatness hypothesis is tested in 2 ways, Pearson’s $\chi^2$ test ($\chi^2$ test) and unbinned Kolmogorov-Smirnov test (K-S test). K-S test has 3 advantages;
1. unbinned, so it has the maximum statistical power
2. no argument with bin choice
3. sensitive with sign change, called “run”

Non of tests shows any statistically significant results.
All data sets are compatible with flat hypothesis, but none of them are excluded either.

<table>
<thead>
<tr>
<th>null hypothesis tests for neutrino mode</th>
<th>low energy</th>
<th>oscillation energy</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>solar</td>
<td>sidereal</td>
<td>solar</td>
</tr>
<tr>
<td># of events</td>
<td>544</td>
<td>420</td>
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<tr>
<td>Pearson’s $\chi^2$:</td>
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<tr>
<td>$N_{d.o.f}$</td>
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<td>107</td>
<td>83</td>
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<tr>
<td>$\chi^2$</td>
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<td>$P(\chi^2)$</td>
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<td>Kolmogorov-Smirnov:</td>
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<td>$P(KS)$</td>
<td>0.42</td>
<td>0.13</td>
<td>0.81</td>
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<td></td>
<td></td>
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<td>0.64</td>
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